Laboratori Nazionali di Frascati

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Measurements of the Rate of Interaction between Stored Electrons and Positrons (*).

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Summary. — The paper describes a series of experiments carried out with the purpose of observing the γ-rays produced in the collision between stored beams of electrons and positrons. The interaction rate has been measured and was found to be in good agreement with the hypothesis that there is a complete overlap between the two beams and that the dimensions of the beams are those calculated from the lifetime effect.

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1. – Introduction.

In this paper we describe a series of experiments carried out with the purpose of observing reactions between electrons and positrons in a storage ring. The storage ring ADA has been described in previous publications (1) and some experiments regarding the lifetime of stored beams and their size have been recently reported (2).

It was felt that our conclusions regarding the size of the beam as well as the mechanism (a coupling of radial and vertical betatron oscillations), which we held responsible for the relatively large height (about 74 μm) of the beam, needed further experimental corroboration.

The overwhelming motive for the research which we report here, was, however, that of finding a monitoring reaction for electrons and positrons, which in the future and with similar but bigger machines (3) must be used as a continuous and instantaneous check of the dimensions of the beam. Such a check is necessary if one wants to obtain absolute measurements for the cross-section of reactions between positrons and electrons.

A monitoring process is defined as a reaction between electrons and positrons which even at the highest energies could be expected to remain unaffected by a possible breakdown of quantum electrodynamics. This restricts the choice to those processes for which in a given geometry the momenta transferred between the interacting particles are small compared with mc. Such processes are:

1) Two-γ annihilations:

$$e^+ + e^- \to \gamma + \gamma \, .$$

For small momentum transfers one has to observe γ-rays at a small angle (with respect to the direction of flight of the colliding positrons and electrons).

2) Small-angle elastic scattering:

$$e^+ + e^- \to e^+ + e^- \, .$$

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(1) C. Bernardini, G. F. Corazza, G. Ghigo and B. Touschek: *Nuovo Cimento*, **18**, 1293 (1960); C. Bernardini, U. Bizzarri, G. F. Corazza, G. Ghigo, R. Querzoli and B. Touschek: *Nuovo Cimento*, **23**, 202 (1962).

(2) C. Bernardini, G. F. Corazza, G. Di Giugno, G. Ghigo, J. Haissinski, P. Marin, R. Querzoli and B. Touschek: *Phys. Rev. Lett.*, **10**, 407 (1963).

(3) The same problem arises in the e^-e^- machines, in particular the Stanford-Princeton double ring now in operation.

This reaction differs from 1) in that the cross-section becomes infinite if the angle of observation tends to zero. The consequent rapid dependence on the geometry of the apparatus makes a quantitative measurement rather difficult.

3) Single bremsstrahlung:

$$e^+ + e^- \rightarrow e^+ + e^- + \gamma \, .$$

The drawback of this reaction is that the interaction of the beam with the residual gas produces a spectrum of γ-rays which is practically indistinguishable from that produced in the reaction.

The observation of a monitoring reaction leads to a determination of what may be called the target volume V. This quantity may be defined in terms of the number densities $\varrho_{\pm}(x, y, z)$ of the colliding positron and electron beams:

$$(1) \qquad N_+ N_- / V = \int \varrho_+(\boldsymbol{x})\, \varrho_-(\boldsymbol{x})\, \mathrm{d}^3 x \, , \qquad \boldsymbol{x} = x, y, z \, .$$

Here x measures the radial distance from the equilibrium orbit, y the distance in the direction of the beam from the ideal point of ecounter and z is the vertical distance from the equilibrium orbit. $N_{\pm}$ are the total numbers of poitrons and electrons.

The target volume can be related to the counting rate by the formula

$$(2) \qquad \dot{n} = f \sigma N_+ N_- L \eta / V \, ,$$

where f is the frequency of revolution of the particles in the beam (in our case $f = 7.45 \cdot 10^7 \ \mathrm{s}^{-1}$) and σ is the cross-section of the monitoring reaction. L is the total length of the beam path viewed by the detector and η is the total efficiency. If L is very small compared to the r.m.s. length of the bunches no correction need be made for the density distribution in the bunches: in this case η is the efficiency of the counter arrangement. In the general case a shape correction η_s defined by

$$(3) \qquad \eta_s = \frac{2}{L} \int_0^{2/L} \exp\left[- y^2 / \lambda^2\right] \mathrm{d}y$$

has to be taken into account. Here λ is the r.m.s. lenght of the bunches $- \varrho \approx \exp[- y^2 / 2\lambda^2]$. At an energy of 205 MeV λ is approximately 6.2 cm.

Since all the parameters which appear in eq. (2) are, with the exception of V, either known or directly measurable, a measurement of the counting rate n will allow one to determine the target volume V.

We have so far investigated reactions (1) and (3). An experiment on reaction (2) is in preparation. The cross-section for (1) is rather small (about 10^{-29} cm^2 at 200 MeV) and our experiments, which represent — with the intensities at our disposal — the limit of what we think can be achieved, are only qualitative (see Appendix). This is aggravated by the fact that process (1) is probably in competition with « double bremsstrahlung » (*i.e.*, $e^+ + e^- \to e^+ + e^- + 2\gamma$), a process which is likely to have a bigger cross-section than (1) at least at high energies; no accurate calculation of the cross-section exists in the literature.

If reaction (3) is used for determining V one has to distinguish between the γ-rays produced in the interaction between the two beams and the γ-rays which are due to the residual gas in the vacuum chamber. A discrimination can be achieved in any one of the following ways:

1) The gas effect should be produced with an intensity proportional to the local pressure all over the circumference of the machine, but the beam-beam effect should be confined to the crossing regions.

2) Changing the density of the beam should change the beam-beam effect but leave the gas effect unaltered.

3) The dependence on the numbers N_+ and N_- is different for the two effects: if the counter arrangement faces, *e.g.*, the electron beam the number of γ-ray counts should vary as N_- for the gas effect but as N_+N_- for the effect due to the collisions between electrons and positrons.

4) The gas effect can be increased by letting the pressure in the vacuum chamber deteriorate. This should not affect the beam-beam interactions.

In the experiments which we report here we have arrived at the following conclusions:

Single bremsstrahlung is a convenient monitoring process. The cross-section which varies only little with energy is so big that even at very low beam intensities quite reliable measurements can be made.

The separation of beam-gas from beam-beam interactions is quite simple and will be irrelevant in bigger machines with higher particle currents.

The target volume determined from our counting rates is in good agreement with the deductions drawn from the observation of the lifetimes of single beams (AdA effect) ([2]).

This means that the two beams can be assumed to circulate in identical orbits. If this were not the case the volume determined from (2) should be larger than that of the AdA effect.

The numerical value of V (about $3.1 \cdot 10^{-2}$ cm^3) is compatible with the outcome of our previous attempts to observe reaction (1).

2. – Supporting data.

In order to determine the target volume V from eq. (2) it is most important to know with the sufficient accuracy the number of positrons and electrons circulating in the ring.

The main source of information about these numbers is given by the readings of the photomultipliers (6 810 A, RCA) which register the synchrotron radiation emitted by the stored particles. Every run was started by looking in turn at the artificially accelerated decay of a small beam of about 100 positrons and 100 electrons. The lifetime of the beam was lowered by overmodulating the radiofrequency. It was always possible to see the disappearance of single particles and it was found that one positron produces a current of about $2\cdot10^{-8}$ A (with a PM voltage of about 2 000 V) an electron about $5\cdot10^{-9}$ A. This difference between electrons and positrons is in part due to a difference in efficiency of the two photomultipliers and in part to a difference in the transparency of the glass windows through which the synchrotron radiation is viewed. The lack of transparency of the electron window is due to the fact that in the way things were arranged the electron part of the equipment was more directly exposed to the « wasted » beam of the Linac.

In order to get a reading at high beam intensities the sensitivity of the photomultiplier is reduced in steps of factors of about 10 (this is done by reducing the PM voltage) till one arrives at a value of about $2.5\cdot10^{-14}$ A/particle at a voltage of about 800 V.

At the end of the run the calibration was often checked by running it in reverse, *i.e.*, by reducing the number of particles by means of an overmodulation of the radiofrequency and by increasing the voltage of the photomultipliers to a point which allowed a direct observation of the disappearance of single particles.

This procedure of calibration has proven very adequate for positrons for which in a space of more than one year we have never observed discrepancies of more than about 10 %. Electrons were much less satisfactory and various checks have revealed the necessity of correcting the calibration by factors of up to about 2, 3. The unreliability of the electron calibration is at least in part due to the fact that the exposure to the Linac beam may induce changes in the dark current of up to about $4\cdot10^{-8}$ A at maximum voltage. Such fluctuations mayv ery strongly affect the first steps in the process of « demultiplication ».

The AdA effect, which has been described in ref. (2) offers a reasonably effective check on the actual numbers of electrons and positrons. Owing to the Coulomb interaction between the particles stored in the same bunch the lifetime of the beam is given by

$$1/\tau = \alpha N + \beta p\,, \tag{4}$$

in which the first term is due to the AdA effect, the second to the interactions (bremsstrahlung and scattering) with the atoms of the residual gas; p is the vacuometer reading. α depends very strongly on the energy ($\sim E^{-5.5}$) and on the r.f. voltage ($U^{-1.5}$). β depends very little on the operating conditions. Measuring N in units of 10^7 particles, p in units of 10^{-9} Torr and with an energy of 206 MeV and 350 W absorbed by the r.f. cavity, we find from an analysis of the decay curves for positrons

$$\alpha = (0.080 \pm 0.004)\,\mathrm{h}^{-1}\,, \qquad \beta = (0.05 \pm 0.02)\,\mathrm{h}^{-1}\,. \tag{5}$$

The high error in β is due to the fact that it has either to be obtained by extrapolating from high beam intensities or by a direct measurement of a lifetime of about 20 h or more. Either method is not very accurate.

In all the runs dealt with in this paper also the electrons were found to decay in accordance with eq. (4). This very strongly indicates that during each run there was no drift in the electron counter and that its readings could therefore be taken to be proportional to the number of electrons. The value of β was always in accordance with (5) but α varied from run to run (presumably for the above-mentioned reasons) from a minimum of about 0.14 to a maximum of about 0.28. The actual number of electrons was therefore assumed to be that which is obtained by applying

$$N_- = \frac{\alpha_-}{\alpha_+} N^0_-\,, \tag{6}$$

in which N^0_- is the number determined by direct calibration. From a comparison of the results of various experiments we estimate the accuracy of this method to be about 10 %.

The only intrinsic information about the nature of the residual gas in the vacuum chamber comes from the vacuometer reading and from a comparison of the measured value of β with theory. The vacuometer readings give a consistent indication of the variation of the pressure, but the absolute value of p has an uncertainty of about 50 % attached to it. Taking account of bremsstrahlung and of the scattering of electrons and positrons by the atomic electrons of the residual gas one obtains

$$\beta = 0.074 P \big(8\langle Z\rangle + \langle Z^2\rangle\big)/128\,, \tag{7}$$

valid for an energy of 205 MeV. Here P is the ratio of the true average pressure in the vacuum chamber to the vacuometer reading (in units of 10^{-9} Torr).

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The first term represents the scattering effect, the second is due to bremsstrahlung. $\langle Z\rangle$ and $\langle Z^2\rangle$ are averages of the atomic numbers for a gas mixture. For pure O_2 the contributions from scattering and from bremsstrahlung are equal.

3. – The coupling coil.

In ref. (2) we had concluded from our lifetime measurements that the beam volume was bigger by a factor of about 60 than what one would expect for a perfect machine in which this volume would be exclusively defined by the interplay of radiation damping and quantum fluctuations. The enlargement of this volume we attributed to a possible coupling between radial (x) and vertical (z) betatron oscillations, which we thought were probably caused by the imperfections of the magnet. We deduced from this hypothesis that it should be possible to increase the lifetime of the beams by introducing an artificial and additional coupling between the radial and vertical betatron oscillations.

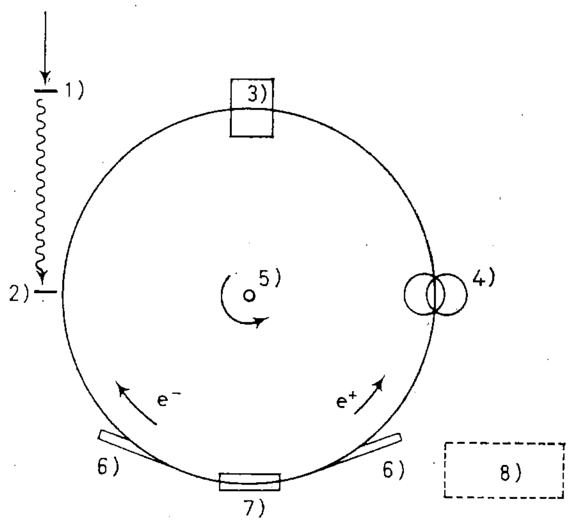

Fig. 1. – Sketch of the ring: 1) radiator for the linac beam, 2) internal converter for injection, 3) r.f. cavity, 4) coupling coil, 5) rotation axis of the whole magnet, 6) synchrotron light exit windows, 7) location of the observed interaction region, 8) location of counter telescope.

To this end a quadrupole coil (compare Figs. 1, 2) was constructed and placed into the quasi-straight sections opposite the injection port of AdA. The measured focal lenght of the coil was about 4.1 m—as compared to the circumference of 406 cm of the orbit — the maximum gradient was 115 G/cm at maximum current.

Lifetime measurements carried out at an energy of 205 MeV and with maximum current in the coil, showed that the decay of the beam was still ruled by eq. (4) and with β in accordance with the estimate (5). The measured

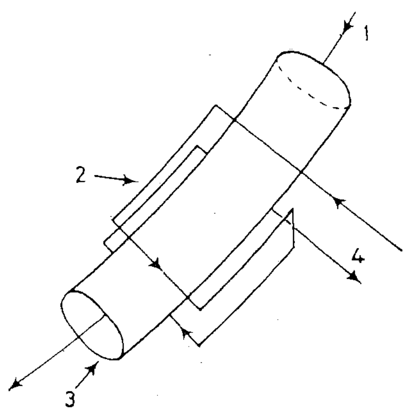

Fig. 2. – Schematic view of the coupling coil. The arrows lie in planes parallel to the median plane. 1) Principal orbit, 2) coupling coil, 3) a section of the vacuum chamber, 4) to power supply.

value of α, however, was 0.035, *i.e.*, smaller by a factor 2.3 than the value obtained with the coil off. It was also verified that this effect was independent of the sign of the current in the coil. The ratio between the α-values determined with and without coil was the same for electrons and positrons.

The simplest interpretation of this effect was of course that the coil (by introducing an additional *r-z* coupling) further increases the volume by a factor of about 2.3. This interpretation is, however, subject to the assumption, that the coil does not affect any one of the other machine parameters, which could influence the value of α. (Compare formula (20) Sect. **6**.2). Indeed there is evidence that the action of the coil is more involved, than this simple interpretation might suggest. A displacement of the beam has actually been observed by taking away the photomultiplier and directly looking at the beam. We have also observed a resonance at about 130 MeV: at this energy and with the coil on, the beam splits into a central part and 3 satellites, the latter with a lifetime of a few seconds. The phenomena connected with the coil are far from being completely understood though the order of magnitude of the effect agrees with what one would predict if one assumes an enlargement of the beam as a consequence of an additional *r-z* coupling. Also — at very low energies (70 MeV) — a vertical broadening of the beam has actually been observed.

The practical importance of the coil consists in that it allows us to postpone saturation in the process of charging the storage ring, since it effectively lengthens the lifetime by a factor 2.3. We have also found (compare Sect. **5**.3) that beam-beam interaction can be significantly reduced by switching on the coil, though the effect is not quite as reproducible as the hypothesis of a simple enlargement of the volume would lead one to expect.

4. – The experimental arrangement.

The reaction $e^+ + e^- \rightarrow e^+ + e^- + \gamma$ is revealed by observing the γ-ray produced in it.

The experimental layout is shown in Fig. 3. S_1, S_2 and S_3 are scintillation counters. S_1 has the dimensions $(10\times4\times1)$ cm³ (horizontal, vertical, thickness) S_2 is $(10\times4\times2)$ cm³ and S_3 $(10\times4\times1)$ cm³. $\check{C}$ is a Čerenkov counter: the lead glass is cylindrical with a diameter of 15 cm and a depth 25 cm and therefore suitable to reveal the γ-rays and to measure their energy. The Čerenkov counter is protected by lead of 5 cm thickness,

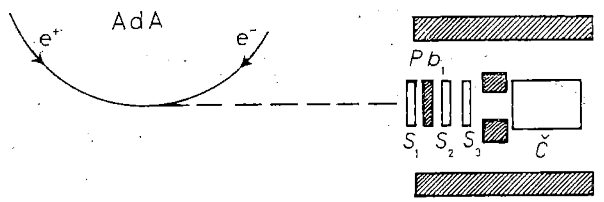

Fig. 3. – Experimental arrangement for the observation of γ-rays from the reactions $e^+ + e^- \rightarrow e^+ + e^- + \gamma$ and $e^+ +$ gas molecule $\rightarrow e^+ +$ gas molecule $+ \gamma$.

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the screening leaving a window of (10×4) cm^2 corresponding to the scintillation counters. A γ-ray from the semistraight Section of AdA is defined by the coincidences S_2S_3C with S_1 in anticoincidence.

The counter S_1 discriminates against incident charged particles. The γ-rays are converted in a lead radiator Pb_1 ($(3.5 \times 9 \times 0.5)$ cm^3) and thus give rise to the coincidence S_2S_3C. The setting of the threshold of the Čerenkov counter determines the range of energy of the spectrum of bremsstrahlung which is registered in our experiment.

The optical axis of the counter telescope coincides to within about 5 mm with the tangent of the electron positron orbit in the crossing point. The distance from this point to the lead radiator Pb_1 is 150 cm. (This was the distance used when the counter telescope faced the electron beam; for positrons we used both 150 and 100 cm).

The coincidences $\bar{S}_1S_2S_3C$ are due to the following four causes:

1) The reaction $e^+ + e^- \rightarrow e^+ + e^- + \gamma$.

2) Bremsstrahlung caused by electrons (if the counter is facing the electron beam) or by positrons (if it is facing the positron beam) in the collision with atoms of the residual gas.

3) Processes, which may take place anywhere in the vacuum chamber and which are caused by the loss of particles mainly in consequence of the AdA effect. In a collision with a particle of the same bunch a particle may gain or lose an energy of a few MeV. If this change in energy is larger than the energy acceptance of the radiofrequency (about 0.35 MeV) the particle will get out of step and hit the inner wall of the chamber after many revolutions. It may then produce a shower the γ-rays of which may trigger the counter arrangement.

4) Cosmic rays.

The way in which processes 1) and 2) may be distinguished will be discussed in more detail in Sect. **5**. The events of type 3) and 4) are eliminated by the following device based on the bunching of the beams in a storage ring. Processes 1) and 2) can only be observed at the time at which the bunches pass the crossing region of the storage ring. Since this region is displaced by 180° from the position of the r.f. cavity, events of the type 1) and 2) can only be observed, when the phase of the r.f. is approximately 0°. (The r.f. works in the 2nd harmonic, the frequency of revolution of the electrons is 73.5 MHz, the frequency of the bunches as well as that of the r.f. is 147 MHz). In processes of type 3) and 4) there will be no synchronization and the events will therefore appear uncorrelated with the phase of the radiofrequency. In this way it will be possible to distinguish the events of class 3), 4) from the events of class 1) 2) by measuring the relative phase between these events and the radiofrequency.

To measure the relative phase we made use of a method similar to the one applied by ANDERSON and MCDANIEL (4). Figure 4 shows a block diagram of the electronic apparatus. The pulses of the counter S_3 are fed to a gate, which only opens on the coincidence $\bar{S}_1S_2S_3C$. The gate output is formed into a pulse suitable for triggering an oscillator of frequency 145 MHz the output of which is mixed with the radio frequency. The 2 MHz beat is then fed into a time-amplitude converter which produces a signal the amplitude of which is proportional to the time which elapses between the receipt of the pulse and the first passage through zero of the beat amplitude. The amplitude of the signal is therefore a measure of the phase of the r.f. at the time of the event $\bar{S}_1S_2S_3C$.

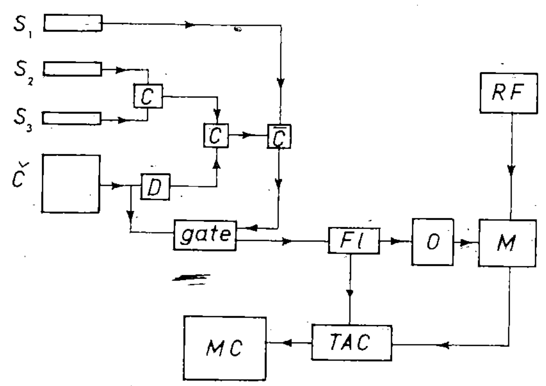

Fig. 4. – Block diagram of the electronics: C, coincidence circuits; $\bar{C}$, anticoincidence; D, pulse discriminator; FI, pulse former; O, oscillator (145 MHz); RF, AdA radio frequency; TAC, time-amplitude converter; MC, multichannel; M, mixer.

The calibration of this apparatus has been carried out in the following way. The vacuum in the doughnut can be made to deteriorate by a factor of up to 100 by switching off the titanium pump. With a pressure approaching 10^{-7} Torr the great majority of all the processes observed will be due to process (2), *i.e.*, to the bremsstrahlung produced in the collision between the particles of the beam and the gas molecules in the crossing region. (Only signals from the crossing region will contribute, since the counter arrangement is directional and the γ-rays produced in the bremsstrahlung from gas atoms are bundled in the forward direction and within an angle of $mc^2/E = 2.5$ mrad, E being the beam energy.)

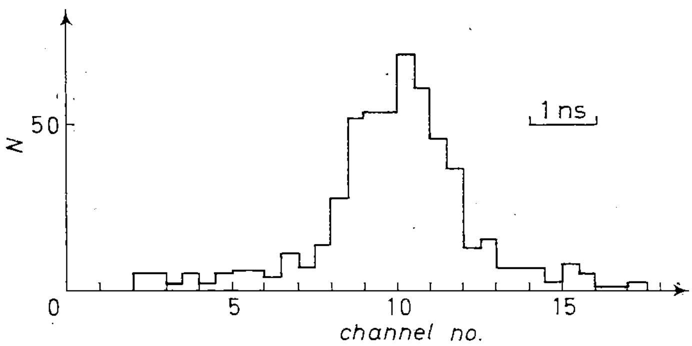

Fig. 5. – Distribution of counts in time in a r.f. period. Bad vacuum in the doughnut.

The spectrum of the time-amplitude converter is then displayed on the multichannel analiser. Such a spectrum is shown in Fig. 5. It is seen that this spectrum is Gaussian with a half-width of about 0.7 ns. This is as good as one

(4) R. L. ANDERSON and B. D. MCDANIEL: *Nucl. Instr. and Meth.*, **21**, 235 (1963).

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can expect if one takes into account the jitter of S_3 as well as the time which elapses between the entry of the head of the bunch into the straight section and the exit of the tail of the bunch. The position of the peak of the spectrum is a measure of where to expect the events coming from reactions 1) and 2).

Figure 6 shows a typical spectrum, which was obtained during a measurement at the usual working pressure (10^{-9} Torr vacuometer reading). It is seen that it is quite possible to distinguish between peak and uncorrelated background and that the number of events in the peak can be determined without significantly increasing the statistical error.

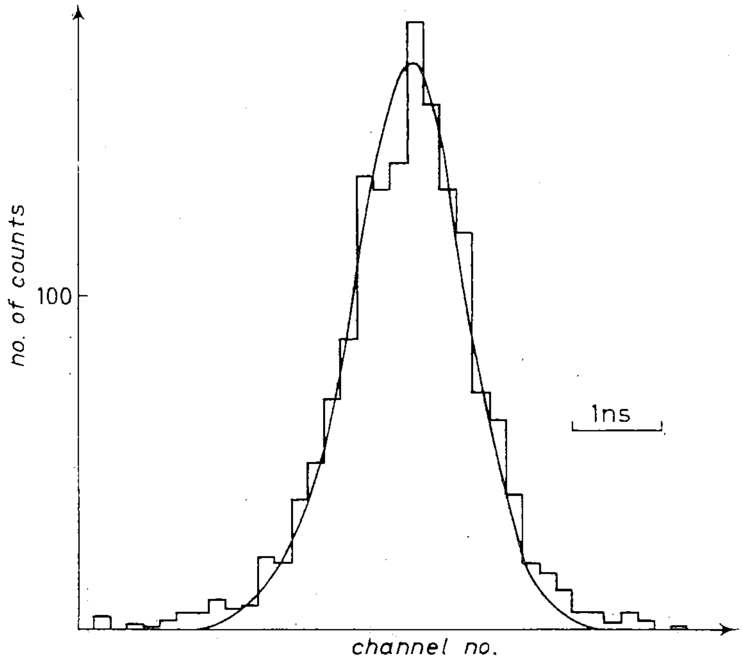

Fig. 6. – Distribution in time of counts in a r.f. period. Good vacuum in the doughnut.

5. – Experimental results.

The following results have been obtained in 4 series of measurements, to which we shall give the numbers 1 to 4 and which were carried respectively in Dec. 63, Jan. 64, Feb. 64 and April 64. These runs had the following characteristics: the energy of the beams was held at 205 MeV in all measurements. The threshold of the Čerenkov arrangement was set in such a way that only γ-rays with an energy of more than 70 MeV were registered. The calibration of this setting was carried out by exposing the counter telescope directly to a very low intensity linac beam (particles arriving at the rate of 1/s).

The calibration has been performed at 50, 100, 150, 200 MeV showing a good linearity and an energy resolution of $\pm 12\%$ at 200 MeV.

In runs 1, 2, 3 the counter telescope was facing the electron beam at a distance of 150 cm. In run 4 it was facing the positron beam first at 150 cm then at 100 cm. The screening was found to be not quite sufficient in December (this made the subtraction of the uncorrelated background difficult): it was substituted by the one described in Sect. **4** and was left unaltered in the runs 2, 3, 4. The coil was installed in January and used in runs 2, 3 and 4. In runs 1, 2, 3 the r.f. system was held on a power level of 330 W, which was reduced in run 4 to 250 W.

In all the runs γ-rays were observed with AdA charged with either one or two beams. In all the measurments the vacuum pressure was checked at intervals of about 10 min.

5·1. *Results obtained with a single beam.* – Figure 7 shows the results obtained in April. The counter was facing the positron beam. The abscissa shows the number of positrons as determined directly from the reading of the photo-multiplier. The ordinate is C_{12}/pN_+ with p in units of 10^{-9} Torr. Here C_{12} is the number of counts in the 12 central channels of the timing display (the whole interval was about 25 channels each corresponding to about 0.28 ns) after subtraction of the uncorrelated background. p is the vacuometer reading. If all the events observed were due to bremsstrahlung produced in the collision of positrons with the atoms of the residual gas we would expect C/N_+p to be a constant.

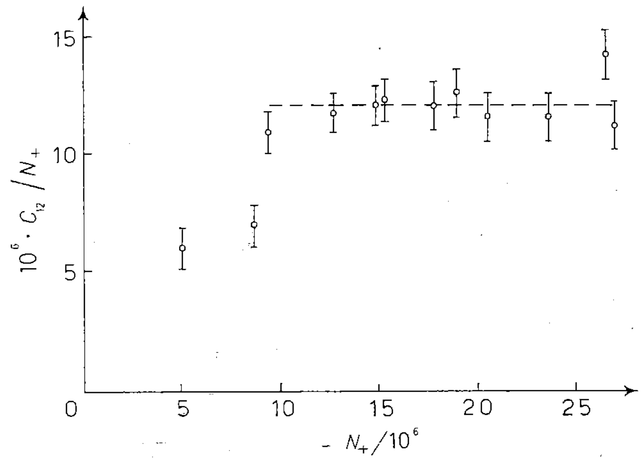

Fig. 7. – Plot of the counts C_{12} in the 12 central channels of the time distribution, divided by N_+ and normalized to $p = 10^{-9}$ Torr. April run only.

It is seen that this assumption holds well in the interval $10^7 < N_+ < 2.5 \cdot 10^7$. We have no explanation for the lowering in the number of counts represented by the two points at $N_+ = 8.6$ and $5 \cdot 10^6$. But we have found no trace of a similar effect in the other three runs. All the measurements with two beams on the other hand were made with $N_+ > 10^7$ (April) and $N_- > 10^7$ (measurements 1, 2, 3). We can therefore conclude that at least in the range covered by the two-beam experiments the single-beam peak counts can be represented by

$$C/N = Ap \,, \tag{8}$$

where A should be a constant and the same for all the runs. The following Table gives the values of A, obtained after the following corrections. All measurements refer to a distance of 150 cm and a corresponding correction has been applied to the April measurements. C refers to the total number of peak counts in 20 min. The number of electrons is determined from eq. (6). No corrections are made for the value of α_+, which is taken to be indicative of the effective r.f. voltage during the run. N in eq. (8) has to be expressed in units of 10^6 particles and p in units of 10^{-9} Torr. The values of α and β reported in the Table refer to the whole run and include also measurements made with two beams.

TABLE I.

Run	α_+	α_-	β	A
Dec.	0.09	0.145	0.08	8.0 ± 1.0
Jan.	0.087	0.162	0.065	5.8 ± 1.0
Feb.	0.078	0.142	0.057	6.7 ± 0.4
Apr.	0.123	0.283	0.043	7.8 ± 0.6

The discrepancies in the values of A between the different runs may be due to various causes. One is the inherent inaccuracy in the measurement of N, another is a possible drift in the threshold of the Čerenkov. Since the integrated cross-section varies approximately with the first power of the cut-off energy this error may be estimated to amount to about 10 %. The weighted average of A deduced from Table I is

$$\langle A \rangle = \frac{7.1 \text{ counts}}{20 \text{ min } 10^6 \text{ particles}} = 5.9 \cdot 10^{-9} \text{ s}^{-1} \text{ particle}^{-1} \,, \tag{9}$$

The deviations of the individual runs from this average, namely $+12\,\%$, $-18\,\%$, $-6\,\%$, $+10\,\%$ are quite compatible with the sources of error, which we have described earlier.

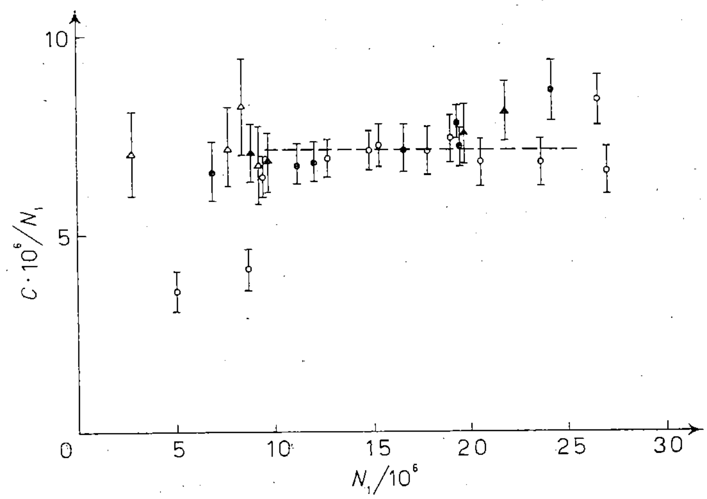

Fig. 8. – Synthesis of all experimental points with a single beam (normalized to $p = 10^{-9}$ Torr).

Figure 8 is a synthesis of all the experimental points obtained with one beam and with $N < 25 \cdot 10^6$. The points have been normalized so as to give the average value $\langle A \rangle = 7.1$ in every run. These points appear to us to confirm very well the hypothesis expressed by eq. (8). In the third run we found a rise in C/N for $N > 2.5 \cdot 10^7$. These points are not shown in Fig. 8, since they were taken very soon after an attempt to degas the vacuometer the readings of which proved rather erratic for several hours.

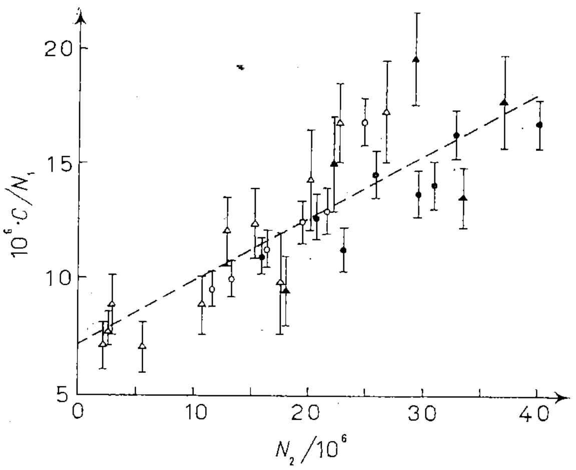

Fig. 9. – Synthesis of all experimental points with two beams. The point at $N_2 = 0$ is normalized to $p = 10^{-9}$ Torr (compare eq. (11)).

Measurements carried out with the counter arrangement facing electrons but with the machine charged only with positrons gave — after the subtraction of cosmic ray events of which we had about 100/h — a total number of counts which was compatible with zero. Indeed the total number of counts was less than 5% of the corresponding number measured with the telescope facing the beam.

5·2. *Measurements with two beams.* – Figure 9 is a synthesis of all the measurements carried out with two beams. It was obtained in the following way. Every single run was fitted by

$$C_i/N_1 = A_i p + B_i N_2 \,. \tag{10}$$

Here C_i is the number of counts obtained in the i-th run in a certain number of central channels in the timing display (this number varied from run to run), N_1 is the number of particles in the beam faced by the counter, N_2 the number of particles in the other beam (in units of 10^6), the numbers representing as before the counts in an interval of 20 min. To make these measurements directly comparable, we have assumed that the term due to the gas gives the calibration of the apparatus. Thus, we have multiplied this number by a factor $\langle A \rangle / A_i$ which we expected to correct for the small differences in the calibration of electrons and the Čerenkov setting. The difference

$$\frac{\langle A \rangle\, C_i}{N_1 A_i} - \langle A \rangle\, p$$

must be further corrected for the April run for the increase of volume — a factor of about 1.07 — due to the lower r.f. power used at that time. The points corrected in this way are plotted in the Figure. In this way every run can be fitted by

$$C_c/N_1 = \langle A \rangle p + B_c N_2 \,, \tag{11}$$

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where C_c and B_c are the corrected values. A best fit then gives for B_c the following values for the four runs: 0.28 ± 0.07, 0.35 ± 0.06, 0.25 ± 0.03, 0.27 ± 0.04. The weighted average of these quantities is

$$B_c = 0.27\pm0.025 \text{ counts/20 min } 10^{12} \text{ particle}^2 = 2.25\cdot10^{-16}\text{ s}^{-1}\pm9\,\% \; . \tag{12}$$

We see that the agreement between the various runs is excellent.

We take this measurement to mean that there is a true beam-beam effect, *i.e.*, that the beams really collide and produce interactions which are registered by the counter-telescope.

To test the validity of this conclusion we consider as an alternative hypothesis

$$C/N_1 = Ap + B(N_1 + N_2) \; , \tag{13}$$

which in physical terms would correspond to the assumption that in some way the residual gas is conditioned by the total number of particles circulating in the machine and in such a way that the local pressure is raised from p to $p + B(N_1 + N_2)/A$. This hypothesis is, however, in contrast with the following facts:

1) The value of B permitted by the measurements with one beam is very small (about 0.04) and therefore insufficient to explain the big effect required by (12).

2) The rates with two beams are effectively dependent not on the sum $N_1 + N_2$ but on N_1 and N_2 separately. For instance, when $N_-=0$, $N_+= 27\cdot10^6$, $C/N_+ = 6.8\pm0.5$; when $N_- = 13.2\cdot10^6$, $N_+ = 11.6\cdot10^6$, $C/N_+ = 10.0\pm0.8$. These data are taken from the April run.

3) The control measurements are reported in the following paragraph.

5·3. *Control measurements.* – As a first control measurement AdA was rotated by about 0.1 rad in such a way that the counter-telescope although still focussed on the tangent of the electron orbit could no longer see the crossing point between the two beams. The results of this measurement (with two beams) are shown in Fig. 10. It is seen that these measurements are in good agreement with $C/N_-p = \varepsilon A = \text{const}$. The efficiency $\varepsilon \simeq 0.5$ is due to the fact that in the point upon which the telescope was focussed the radius of curvature is smaller by about a factor 2 than in the centre of the « quasi-straight » Section. The dashed line represents the best fit $C/N_- = \varepsilon(pA + BN_+)$ obtained from the measurements with two beams in which the counter-telescope was pointing at the centre of the interaction region. We retain this control to represent the most direct proof of the assumption that the quantity $C/N_- - Ap$

discussed in the Sect. 5·2 is indeed due to the effect of collisions between electrons and positrons.

As a second control we have carried out measurements of the counting rate, in which the quadrupole coil described in Sect. 3 was switched on. As we have explained in that Section we can only attribute a qualitative significance to this check, since the action of the coil is not perfectly understood. The measurements of B with the coil on have given the following values, using the same corrections as in the preceding paragraph: 0.09 ± 0.09 for January, 0.19 ± 0.03 in February and 0.07 ± 0.03 in April, with an average of 0.15. Apart from the large fluctuations from run to run, which we do not understand, there is clear evidence that the beam-beam effect is reduced by an average factor of 1.8 if the coil is switched on. Since the coil does in no way affect the measurements with a single beam (4 points in Fig. 8 were taken with the coil on) we interpret this reduction as further evidence of the assumption that $CN_- - Ap$ is due to an interaction between the two beams.

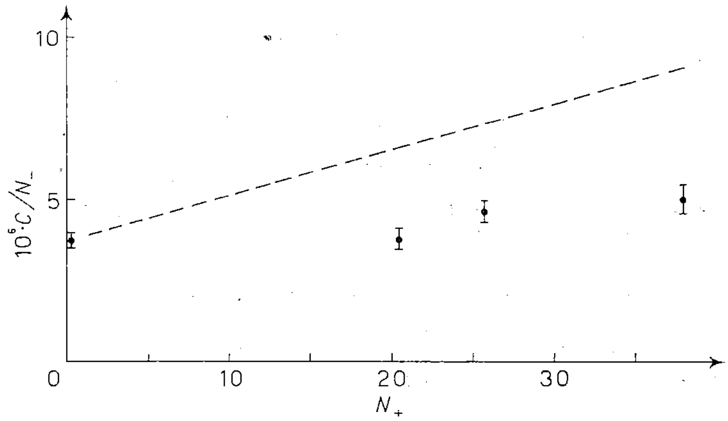

Fig. 10. – Results of the control measurements (counts from a noncrossing point, with two beams).

6. – Interpretation of the experimental results.

In this Section we want to establish the relation between the results of the measurements with one and two beams, expressed in terms of the numbers $\langle A \rangle = 5.9 \cdot 10^{-9}\ \text{s}^{-1}$ and $B = 2.25 \cdot 10^{-16}\ \text{s}^{-1}$ and the supporting data expressed in terms of the numbers $\beta = 0.05\ \text{h}^{-1}$ (the limit of the decay constant of the beams for low intensity and at a pressure of 10^{-9} Torr) and $\alpha = 8 \cdot 10^{-9}\ \text{h}^{-1}$ (the constant of the AdA effect). The parameters $\langle A \rangle$ and β depend on the properties of the residual gas and are independent of the beam density, the parameters B and α are indipendent of the properties of the gas and inversely proportional to the volume of the beam. A and β can be determined from measurements with a single beam and a comparison of these two numbers can therefore be carried out by using the single-beam results only. The main physical process involved in A is bremsstrahlung from the gas atoms, producing a γ-ray of energy > 70 MeV. The physical processes involved in the determination of β are: bremsstrahlung from gas atoms producing γ-rays with an energy greater than the energy acceptance of the r.f. (about 0.35 MeV), scattering from the atomic electrons in which the energy loss is bigger than 0.35 MeV. A discussion of the results obtained with one beam only is therefore equivalent to a comparison of these physical processes.

6·1. *Interpretation of the results obtained with a single beam.* – Since the counter-telescope was placed at a distance of 150 cm from the centre of the straight section, and since the radius of curvature in the «straight» section was also about 150 cm one should expect that the counter would view a length $L = 9$ cm (*i.e.*, the width of the converter) of the crossing region. From this length we should subtract about 4 mm to correct for the finite aperture (about 2.5 mrad) of the γ-rays emitted in the process of bremsstrahlung. We therefore assume $L = 8.6$ cm and we expect that A be given by

$$A = \eta (L/u) c \sum_Z n_Z \sigma_Z \,, \tag{14}$$

where $u = 404$ cm is the circumference of the equilibrium orbit, n_Z is the density of atoms of species Z in the residual gas at a pressure of 10^{-9} Torr, σ_Z is the cross-section for the production of a quantum with energy > 70 MeV from such an atom and η is the nongeometrical part of the detection efficiency.

Taking account of screening but neglecting the variation of the screening parameter under the logarithm one can write for the cross-section

$$\sigma/Z^2 = 1.04 \cdot 10^{-26} \text{ cm}^2 \,. \tag{15}$$

We can define $p_1\langle Z^2\rangle$ by the relation

$$\sum_Z n_Z \sigma_Z = p_1 \langle Z^2 \rangle 7.4 \cdot 10^{-19} \text{ cm}^{-1} \,, \tag{16}$$

where p_1 is the true pressure in the crossing region, when the vacuometer reads 10^{-9} Torr. Inserting for A from eq. (9) we then get from (16)

$$\eta p_1 \langle Z^2 \rangle = 12.6 \,. \tag{17}$$

The efficiency η can be assumed to consist of two factors. The first represents the probability of conversion in the lead converter. Since this converter is about 1 radiation length in thickness this factor should be approximately 0.55 (*). Counts may be lost as a consequence of the passage of the γ-rays through the seam of the vacuum chamber. In the average we expect the γ-rays to pass

(*) The conversion length is nearly 0.8 of a radiation length.

approximately 6 mm of stainless steel, which with a radiation length of 1.83 cm makes one expect an attenuation of about 22%. We therefore estimate η to be $0.55(1-0.22)=0.45$, so that from (17) we deduce $p_1\langle Z^2\rangle=28$. This has to be compared with eq. (7). If one assumes pure O_2 as the prevalent constituent of the residual gas one gets $P=0.66$, P representing the average pressure in the beam path (corresponding to a vacuometer reading of 1 in units 10^{-9} mm Hg). This has to be compared with $p_1=0.44$. Taken at its face value this result would mean that the average pressure in the chamber is about 50% higher than the local pressure at the point viewed by the counter telescope. This is not at all unreasonable if one considers that the pressure reading usually deteriorates by a factor of about 1.6 if the r.f. is switched on (compare Fig. 1). If one assumes that the residual gas consists of a mixture of, say, O_2 and H_2 in equal proportions one obtains from (7) $P=1.25$ and from (17) $p_1=0.85$ and it is seen that the ratio between local and average pressure does not change appreciably.

Considering the large errors attached to the value of β we therefore conclude that the counting rates of our measurement are compatible with the information about the gas obtained by lifetime measurements and that there is an indication that the local pressure is slightly lower than its average value.

6·2. *Interpretation of the results obtained with two beams.* – The effective length viewed in the experiments with two beams and normalized to the geometry applied in the runs 1, 2, 3 is defined as $L_{\text{eff}}=\eta_s L$ with η_s given by eq. (3). In this way one obtains $L_{\text{eff}}=7.6$ cm.

The integrated cross-section for the production of γ-rays in electron-positron collisions is known ([5]). Taking the limits of integration to be 70 MeV to 205 MeV one has

$$\sigma_0 = 2.7\cdot 10^{-26}\ \text{cm}^2\,. \tag{18}$$

The factor between (18) and (15) can be understood if one remembers that electron-positron scattering is not screened; one would therefore expect that $\sigma_0\langle Z^2\rangle/\sigma=\log\gamma^2/\log 183\,Z^{-\frac{1}{3}}$ and this is approximately true.

Inserting from (18) into (2) and using the average value 0.27 for B we obtain

$$V/\eta = 7\cdot 10^{-2}\ \text{cm}^3 \tag{19}$$

Using the value $\eta=0.45$ determined in the previous Section this gives $V=3.1\cdot 10^{-2}$ cm³.

This volume has to be compared with the volume which can be determined from the AdA effect. It has been shown previously that the coefficient α can

([5]) G. ALTARELLI and F. BUCCELLA: *Thesis* (unpublished).

be calculated if one assumes that the disappearance of particles is entirely due to the Coulomb scattering within the same bunch. In this way one obtains (2)

$$\alpha = \sqrt{\pi}\, r_0^2 c (mc^2/\varepsilon)^2 (mc/q) L(x)/kV\,, \qquad x = \gamma q c/\varepsilon\,. \tag{20}$$

Here $k = 2$ is the harmonic number, $r_0 = 2.8 \cdot 10^{-13}$ cm the Lorentz radius of the electron, ε the « central » energy acceptance of the r.f., q the r.m.s. momentum of the radial betatron oscillations and γ is the energy in units of mc^2. For the factor L we have in good approximation

$$L(x) = \log x - C/2 - \tfrac{3}{4}\,, \tag{21}$$

(where C is the Euler constant).
Inserting into this equation the value of α given in (5) and putting

$$q/mc = 0.159\,, \qquad \varepsilon/mc^2 = 0.80\,, \tag{22}$$

values which follow from the characteristics of AdA at an energy of 205 MeV, a momentum compaction of 2.2 and a r.f. voltage of 5.5 kV, we get

$$V = 3.1 \cdot 10^{-2}\ \text{cm}^3\,. \tag{23}$$

The coincidence with the value of V deduced from bremsstrahlung, rests on the determination of the r.f. voltage. About this voltage we have the following data. The cavity absorbed an average of 330 W. Its shunt impedance was measured before the vacuum chamber was baked out and found to be $> 30\,000\ \Omega$. This means that $U > 4.45$ kV. A direct check of the lifetime as a function of the absorbed power had given $U = (5.3 \pm 0.3)$ kV. An accurate measurement of the shunt impedance was carried out with a twin cavity and gave $45\,000\ \Omega$ corresponding to $U = 5.45$ kV.

It is therefore seen that the value determined from the observation of bremsstrahlung is in excellent agreement with the value determined from the AdA effect. This value, *i.e.*, $3.1 \cdot 10^{-2}$ cm^3, implies the following dimensions for the bunches. Defining as the height h of the beam $2\sqrt{\pi}$ the r.m.s. value of the vertical density distribution and putting $V = awh$, where w is the width ($2\sqrt{\pi}$ r.m.s.) and a the total length of the bunches, we have

$$a = 22\ \text{cm}\,, \qquad w = 0.19\ \text{cm}\,, \qquad \text{h} = 0.0074\ \text{cm}\,. \tag{24}$$

The effective cross-section of the beam is $S = 0.00141$ cm^2.

* * *

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APPENDIX

The inconclusive outcome of our attempts to observe annihilation radiation.

Our first attempts to observe beam-beam interactions had been to reveal the γ-rays produced in the two-γ annihilation

$$e^+ + e^- \to 2\gamma .$$

Since the time of the first determination of beam size via the AdA effect (2) we knew that the counting rate for this reaction was prohibitively small. We can now justify the negative results with the data we dispose of after the bremsstrahlung experiment.

The experimental arrangement consisted of two lead glass Čerenkov counters (plus anticoincidence scintillation counters) facing respectively the electron and positron beam. Coincidences between the two counters were registered and analysed with respect to pulse size and timing relative to the phase of the radio frequency.

The γ-rays produced as annihilation radiation are as in the case of bremsstrahlung emitted into a narrow forward (and backward) cone relative to the beam. The angular distribution is a little less favourable than in the case of bremsstrahlung falling off less rapidly at high angles. At our beam energy the total cross-section should be about 10^{-29} cm^2, which is 2700 times smaller than the cross-section for bremsstrahlung. From this and correcting for the actual geometrical conditions of this experiment we conclude that given the counting rate for bremsstrahlung the counting rate for two-γ annihilation should have been

$$\dot{n}_{2\gamma} = 5 \cdot 10^{-16} N_+ N_- \,\text{h}^{-1} . \tag{25}$$

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The total number of possible annihilations observed in about 4 h of measurement and with an average $\langle N_+ N_- \rangle$ of about $5 \cdot 10^{14}$ was 2 ± 2. This is quite compatible with the expected counting rate (25).

We also had observed a number of coincidences in which the pulse heights of the two signals were significantly different from one another, their sum being less than what would correspond to the total energy of 410 MeV liberated in the annihilation. The present discussion shows that these events cannot be attributed to annihilation and bears out our conjecture of having observed a different process, probably double bremsstrahlung.

RIASSUNTO

Si descrive una serie di esperimenti allo scopo di osservare i raggi γ prodotti nelle collisioni tra fasci accumulati di elettroni e positroni. È stata misurata la velocità di reazione che risulta in buon accordo con l'ipotesi di completa sovrapposizione dei due fasci all'incrocio; inoltre le dimensioni dei fasci sono quelle stesse che si calcolano a partire dall'effetto di vita media.